Performance measurement is not a simple matter. There exist different methods which are using arbitrary hypothesis. These methods are based on actuarial calculations to calculate present values.

Two methods are presented here: they differ according to who is the main actor, the investor or the fund manager. These two methods converge only if there is no intermediate flow.

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1 – Performance rate

Definition: a financial valuation is a monetary amount attached to a portfolio or to an asset/liability, at a specific defined date.

Definition: the performance gap is the difference between the final valuation and the initial valuation, expressed in monetary unit. It's also a profit and loss (PL).

Definition: the performance is a relative indicator that relates a valuation recorded at an initial date, to a valuation recorded at a final date.

The performance is measured over a duration (or a period), either as a rate, either as an index:

- Measure in rate (%), as the ratio of the performance gap over the initial value.
- Measure as an index: setting the index value at 100 at an origin date, and calculating the relative variation of this index over the period.

When the performance is measured in rates, two forms occur:

- The cumulative performance, measured in percentage (%).
- The periodic performance, usually annualized, measured in percentage per year (% /year).

The cumulative performance is gross, it represents the change in value throughout the reporting period.

Annualized performance is reported year-round, either on a calendar year or on a constant matury basis.

The rate of return is a measure in rate of the performance.

2 – Cumulative Performance

Measuring cumulative performance

With P0, the valuation at the initial date T0 and P1, the valuation at the final date T1.

The PL is: P1 – P0

The Performance is:

- With a **arithmetic rate: R_A%**,

$$R_A\% = \frac{P_1 - P_0}{P_0} = \frac{P_1}{P_0} - 1 \qquad (2.1)$$

- With a **Geometric rate: R_G%**:

$$R_G\% = Ln\frac{P_1}{P_0} \qquad (2.2)$$

Ln: the logarithm.

The arithmetic rate is simpler to use, but it generates a dissymmetry which does not exist with the geometric rate: an increase followed by an identical decrease produces 2 different arithmetic rates, whereas the geometric rates are only opposite (change of sign).

> Example: valuation on 3 dates: 100, 110 and 100.
> The performance is a null over the full period (from 100 to 100).
> For each sub period:
> The arithmetic performances are: 10% and -9.09%
> The geometric performances are: + 9.53% and-9.53%
> We geometric rate has a much better additivity than the arithmetic rate.

Profitability with intermediate flows of investment

In the case of intermediate flows, between the initial date and the final date, it is possible to simplify calculations with a few additional hypotheses.

In the case of intermediate flows **of investment** (Cash input or cash withdrawal), there are two possibilities to assign the cash flow:

- At the initial date:

  P0 becomes (P0 + F)

- At the final value:

  P1 becomes (P1 – F)

In the case of intermediate flows **of income** (dividends), we use the same convention as before, either initial or final positioning of the flow, but we must change the sign of the flow: F becomes (– F).

- Initial date with investment flows F> 0

$$R_A\% = \frac{(P_1)-(P_0+F)}{(P_0+F)} \quad (2.3)$$

- Initial date with disinvestment flows F< 0

$$R_A\% = \frac{(P_1)-(P_0-F)}{(P_0-F)} \quad (2.4)$$

- Final date with investment flow F> 0

$$R_A\% = \frac{(P_1-F)-(P_0)}{(P_0)} \quad (2.5)$$

- Final date with disinvestment F< 0

$$R_A\% = \frac{(P_1+F)-(P_0)}{(P_0)} \quad (2.6)$$

With the same model, for revenue streams by changing the sign of F:

- Initial flow with income stream F> 0

$$R_A\% = \frac{(P_1)-(P_0-F)}{(P_0+F)} \quad (2.7)$$

- Final flow with income streams F> 0

$$R_A\% = \frac{(P_1+F)-(P_0)}{(P_0)} \quad (2.8)$$

The latter case corresponds to the definition of the Total Shareholder Return, in the case of a dividend distribution:

$$TSR\% = \frac{P_1-P_0+Dividend}{P_0} \quad (2.9)$$

Stock Exchange method - index.

The performance for stocks can be calculating the performance:

$$(1+p\%) = \frac{End\,Valeur}{Start\,Valeur}.\prod_{k=1}^{n} Coeff(k) \quad (2.10)$$

This method allows performance adjustments due to corporate actions, such as:

- Share Split and Share Reverse Split
- Bonus Issue, and rights issue
- Dividend

The calculation of each coefficient occurs at the date of the event:

$$Coeff(t) = \frac{Value(t)+Flow}{Value(t)} = (1+\frac{Flow}{Value(t)} \quad (2.11)$$

For example, in the case of a dividend distribution, the expression of the adjustment coefficient is as follows:

$$Coeff(t) = (1+\frac{Dividend}{Value(t)}) \quad (2.12)$$

This method is used to calculate indices.

Multi-time Performance chaining

Performances must be compounded over several periods through consistent formulas that make additivity of performances correct.

Arithmetic performances are compounded **with multiplication** of capitalization factors: (1 + R%)

$$(1+R_A\%) = \frac{P_T}{P_0} = \frac{P_1}{P_0}\dots\frac{P_T}{P_{T-1}} = \prod_{k=0}^{T-1}\left(1+\frac{P_{k+1}-P_k}{P_k}\right) = \prod_{k=0}^{T-1}(1+r_k) \quad (2.13)$$

Geometric performances are compounded with **addition** of single period rates:

$$R_G\% = Ln\left(\frac{P_T}{P_0}\right) = \sum_{k=0}^{T-1} Ln\frac{P_{k+1}}{P_k} = \sum_{k=0}^{T-1} r_k \quad (2.14)$$

The compounding formula is much simpler with geometric rates.

3 – Annualized Performance

Cumulated performances cannot be compared between each other because their underlying periods are different. Annualized performances are standardized over a year to allow comparisons.

Definition: the annualized performance is equal to the cumulative performance brought back to a normalized duration of one year.

The annualized performance is homogeneous to an interest rate: " % Per year "(T^{-1}).

Annualized performances introduce the time T in the calculation formulas: the cumulative rate R%, transforms into annualized rate: R.t

The introduction of the duration T, introduces an new method we called: the actuarial method.

- Annualized arithmetic performance:

$$(1+r_A.T) = \frac{P_1}{P_0} \quad (3.1)$$

- Annualized actuarial performance:

$$(1+r_G)^T = \frac{P_1}{P_0} \quad (3.2)$$

- Annualized geometric performance:

$$(e)^{r_c T} = \frac{P_1}{P_0} \quad (3.3)$$

The duration T is defined in years: It's a decimal number, with T = 1 for a full year.
The methods differ in the way of transforming the ratio P1/P0 into annualized rates that take into account the duration T of each invested period.

Compounding of rates

Arithmetic annualized performance chaining:

$$(1+r_A.T)=\prod_{k=1}^{m}(1+r_k t_k) \quad (3.4)$$

Actuarial annualized performance chaining:

$$(1+r_G)^T=\prod_{k=0}^{m}(1+r_k)^{t_k} \quad (3.5)$$

Geometric performance chaining:

$$r_c T=\sum_{k=0}^{m} r_k t_k \quad (3.6)$$

Global transformation formula :

$$P_1=(1+r_A.T)P_0=(1+r_G)^T P_0=e^{r_c T}P_0 \quad (3.7)$$

This formula allows you to transform one rate into another rate.

4 – Performance for investors

We present methods for calculating annualized rate of performance in the case where there are multiple cash-flows during each period. The methods will be rather different depending on weather we are investor or fund manager.

IRR method: Internal rate of return

IRR is a performance measure for investments with various capital inflows and outflows over time.
The performance R, is the rate that cancels the present value of all cash-flows:

$$\sum_{k=0}^{T}\frac{F_k}{(1+r)^{t_k}} \quad (5.1)$$

With F_K: Cash-Flow on date t_k
F_0 : Initial cash-flow and F_T: final cash-flow.
These cash-flows are either positive or negative. A necessary condition for this equation to have a solution is to have inflows and outflows.
In order to get a rigorous IRR calculation, some conventions must be fulfilled:

- The initial cash-flow is viewed as a purchasing transaction with an amount equal to the market value: this is the initial valuation of the portfolio
- The final cash-flow is viewed as a saling transaction with an amount equal to the market value: this is the final valuation of the portfolio
- Intermediate cash-flows are the cash-flows of investment or divestment
- The revenue cash-flows are either reinvested and then are taken into account in the final value, or are distributed and are taken into account as intermediate cash-flows.

Remark: IRR calculation does not require intermediate valuations. The IRR method is a performance method for investors.

5 – Performance Manager

Dietz Method

The Dietz method starts by splitting the total period into sub-periods without intermediate cash flow. The method is then divided into two steps:

- Calculate a cumulative performance on each sub-period with the following formula to get a periodic rate :

$$1+R_A\%=\frac{P_{end}}{P_{start}} \quad (6.1)$$

$P_{Beginning}$: valuation at the beginning of the sub-period taking into account all transaction and flows from the previous periods.
P_{End}: valuation at the end of the sub-period, taking into account the revenue cash-flows reinvested, but not the investment cash-flows.

- Compound all the periodic rates to cover all the period. This way, we get R, a global cumulative rate,

$$(1+R)=(1+R_1).(1+R_2)\dots(1+R_T) \quad (6.2)$$

This method becomes independent of the intermediate cash-flows. The performance of the manager does not depend on external investment cash-flows.
On the other hand, the method requires a valuation at each intermediate date with a net asset value calculation.
The Dietz method is a performance method for fund managers.

6 – Comparison

These two methods are fundamentally different and should not be confused. They produce identical results only if there is no intermediate flow: the investor keeps the same level of investment over the entire investment period.

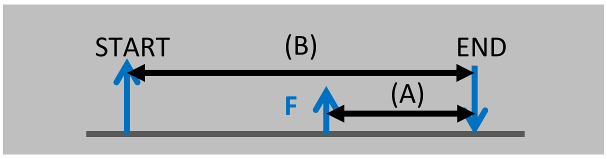
The modified Dietz method.

This method is used to adjust the classic Dietz method, taking into account intermediate fluxes, such as the sorting method.

$$R\% = \frac{P_{fin} - P_{début} - \sum_k F_k}{P_{début} + \sum_k w_k F_k} \qquad (7.1)$$

With F_k: intermediate Flow

And w_k: weighting factor of the duration:



$$w_k = \frac{number\ of\ days\ with\ F}{number\ of\ days\ in\ period} = \frac{(A)}{(B)} \qquad (7.2)$$

The benefit of this method compared to the classic Dietz method is that it does not require intermediate valuation. An algebraic transformation of this formula gives the equivalence:

$$V_{end} = (1 + R\%)(V_{start}) + \sum_k (1 + w_k R\%). F_k \qquad (7.3)$$

This Formula Is Identical to the IRR formula, but with proportional rates rather than with actuarial rates. In conclusion, despite its name, the Modified Dietz method is a performance method for investors.

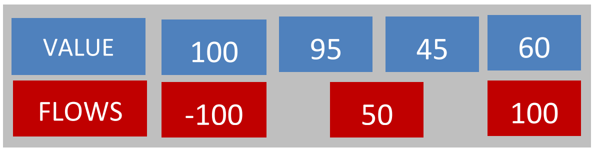
In the financial literature, the previous methods are also named: TWRR and MWRR

- TWRR: Time Weighted Rate of Return - this is the Dietz method.
- MWRR: Money Weighted Rate of Return- this is the IRR method.

7 – Application example

Performance over three months:

Initial fund Value: 100
Value at 60 days before redemption: 95
Disinvestment at 60 days: 50
Value at 60 days after disinvestment: 95-50 = 45
Value at 90 days: 60



Solutions:

- IRR method on (-100; 0; 50; 60)
  IRR 1 month: 3, 82% IRR 3 months: 11.91%
- Modified Dietz Method: 10/83.3 =12.00%
- Dietz method: 0.95x1.33 = 1.2667 then 26.67%

Question: why the asset manager performance is twice that of the investor performance?

Answer: because the investor invests 100 for 2 months with negative performance and invests 50 the last month with a positive performance. The asset manager invests over the full three months.

8 – Conclusion

One must be very careful when using these calculations. There are many methodological choices, between the types of rates and the methods. Depending on the use, the results will be different. The Modified Dietz method for investors is simpler than the IRR, because it avoids the numerical complexity of the IRR. The Modified Dietz method provides a arithmetic rate, while the IRR provides a geometric rate, the choice is yours.

-/-/-